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1 Goodman's New Riddle of Induction

Nelson Goodman has shown that a theory of induction requires a criterion of lawlikeness. Assume a qualitative concept of confirmation for simple universal hypotheses, such as "All A's are B's" ($\forall x(A(x) \Longrightarrow B(x))$). Goodman uses the following terminology:

1. A simple universal hypothesis is called **non-exhausted** if and only if not all objects which satisfy the antecedent condition $A(x)$, have been checked or examined with respect to the consequent condition $B(x)$.

2. All simple universal hypotheses are called **violated** or **falsified**, if and only if at least one object, say e, has been discovered such that $(A(e) \land \neg B(e))$ holds. We could also say that e is a **negative single case**.

3. By a **positive single case** of a universal hypothesis we understand an observed individual b, which satisfies the antecedent as well as the consequent, that is, $(A(b) \land B(b))$ holds.

4. If o does not satisfy the antecedent, that is, $A(o)$ is false, then o is neither a **positive single case** nor a **negative single case**. Be clear as to why this is so.

Clearly for prognostic considerations, only non-exhausted hypotheses are of interest. Exhausted hypotheses can only exist in finite cases. We are not interested in falsified hypotheses; we cannot make further predictions with them. For the *potential* confirmation of a hypothesis H it is not significant whether a positive single case has been obtained. One can only then speak of an actual confirmation of H when at least one positive single case has been discovered.

Assume that H is a non-exhausted and non-violated simple universal hypothesis for which there exist positive single cases. We cannot yet say on the basis of these assumptions that H is inductively confirmed. To say that H is inductively confirmed, would presuppose that it is reasonable to assume that the unexamined cases that satisfy the antecedent condition, will also satisfy the consequent condition. Goodman maintains that when H is a lawlike proposition such an assumption is reasonable enough. For example consider the proposition:

H_1: All copper conducts electricity.

A few positive cases would confirm H_1. That is, a number of positive cases increase the plausibility of the assertion that other objects which consist of copper also conduct electricity. If on the other hand a hypothesis is an accidental proposition, then such an inference from verified to unverified single cases does not seem to be justified.

The transition from examined to unexamined cases appear reasonable when H constitutes a lawlike proposition and unreasonable when H is an accidental universal proposition.

The usual theories of confirmation do not came to terms with these problems because they do not differentiate between these two types of propositions. According to Goodman, a solution of the problem of induction presupposes the formulation of a criterion of lawlikeness. As long as one does not possess such a criterion one must either depend on vague and intuitive notions of lawlikeness, or one denies the necessity for a clear differentiation between lawlikeness and accidental propositions. But then all kinds of propositions must be treated equally with respect to confirmability and we must subsequently accept very implausible results. For example, the observation of a few red cats in city X would confirm the non-falsified hypothesis that "all cats in city X are red".

But Goodman showed more. If one denies the difference between lawlike and accidental propositions, or if one admits both as equally suitable candidates for inductive confirmation, then one is not only confronted with implausible results, but also with logical contradictions. We shall illustrate this with two examples. The first example is analogous to Goodman's chosen example.

Let the predicates E and G be abbreviations of "emerald" and "green" respectively. We shall first consider the following proposition.

$$\forall x(E(x) \Longrightarrow G(x)) \quad \text{(All emeralds are green)} \tag{1}$$

We require that the "colour green" is not part of the definition of the concept "emerald". Therefore, we can consider (1) to be an empirical hypothesis. Since until today only green and no non-green emeralds have been found, and many emeralds have not yet been discovered, we can consider (1) to be an unexhausted, non-violated hypothesis with individual positive cases. Since the individual positive cases are large in number, we consider the hypothesis (1) well confirmed. The assumption that "all emeralds that will be discovered shall also be green," appears on this basis to be highly plausible. However, as long as we do not differentiate between lawlikeness and non-lawlikeness we can establish the following paradox: The assumption

A^*: All emeralds found in the future shall be blue.

is as well confirmed as is the assumption

A: All emeralds found in the future shall be green.

A^* is equally well confirmed as A on the basis of the very same past observations of green emeralds which confirm hypothesis (1). In order to prove this we only need to introduce a suitable predicate and substitute it for the predicate in the consequent. Let us call this new predicate "grue" (G^*). We define G^* as follows:

Definition 1.1 $G^*(x) \stackrel{def}{=}$ *x is observed with respect to colour before t_0 and x is green,* ***or*** *x is not observed with respect to colour before t_0, and x is blue.*

Take notice of the exact statement of the definiens of the predicate "grue". Specifically, a proposition, according to which an object a is grue, does not imply or constitute the assertion of a colour change of a at t_0 namely, before t_0, a is green, after t_0, a is blue. If a was observed before t_0 and was seen as green, then a is grue. And a is grue even if a remains green after t_0 until, say, the end of its existence. And if a was not examined before t_0 as to its colour, then a is grue if and only if a is blue before t_0 as well as after t_0. We assume here that colour properties are properties that last over a reasonable amount of time in order to avoid unnecessary complications.

Our original hypothesis (1) becomes

$$\forall x(E(x) \Longrightarrow G^*(x)) \quad \text{(All emeralds are grue).} \tag{2}$$

Several things can now be asserted:

1. All hitherto observed positive single cases of (1) are also positive single cases of (2). That is, until this moment all observed emeralds are green and hence by definition are grue. By definition all emeralds that are examined before t_0, and are found green are characterized as grue. Hence with respect to the confirmation through positive single cases, (2) must be considered to be equally well confirmed as (1).

2. However, future expectations based on (2) are logically incompatible with those based on (1). If we accept the hypothesis that "all emeralds are green" then we must be convinced that all emeralds found in the future will be green. If on the other hand, we accept that all emeralds are grue, then we must be convinced that all emeralds to be discovered in the future will be grue. Now an emerald found and examined for its colour after t_0 is, by definition, only then grue, if it is blue.

Hence we obtain the paradoxical result that the same numerous past observations of green emeralds appear to confirm the hypothesis

(a) All future found emeralds will be green

as well as the incompatible hypothesis that

(b) All future found emeralds shall be blue.

Hence through the introduction of suitable predicates it is possible to create as many incompatible future expectations about the colour characteristics of emeralds as there exist different colour predicates. Moreover, it should be clear by now that with the same procedure and suitable predicates, it is also possible to support arbitrary future expectations on the basis of observed past regularities.

This paradox remains, according to Goodman, as long as it is not possible to introduce a criterion — independent of the concept of confirmation — which differentiates between propositions capable and not capable of confirmation. In ordinary practice one escapes this paradox on the basis of an intuitive, imprecise differentiation between lawlike and accidental propositions. Hence only (1) is a lawlike hypothesis, whereas (2) is not. Hence we consider proposition (1), but not proposition (2), to be capable of empirical confirmation. That is, having observed positive cases, we believe in the correctness of (1) but not of (2). With this reaction we admit that the crux of the problem is the differentiation between lawlike universal propositions capable of confirmation, and accidental-universal propositions not capable of confirmation.

Goodman calls a predicate like "green", **projectable**, whereas predicates such as "grue" are non-projectable or non-transferable. For projectable predicates it seems reasonable to inductively project from observed events to unobserved future events. For non-projectable predicates this would seem unreasonable. Hence instead of searching for a criterion to distinguish between lawlike and hence inductively confirmable propositions, and those that are non-lawlike and hence non-confirmable propositions, one could search for a criterion to differentiate between projectable and non-projectable predicates. On this basis (2) is accidental and not lawlike, because the predicate of the consequent is not projectable. Intuitively one could say: Crazy predicates like "grue" are unsuitable for the construction of confirmable hypotheses. Since "green" is projectable, "All emeralds are green" lends itself to inductive confirmation, whereas, "All emeralds are grue" does not.

It is clear that "green" and "grue" differ in one important respect, and one could argue: The predicate grue was defined with reference to a specific temporal coordinate t_0 which explicitly enters into its meaning. One might say that such predicates are not projectable and are therefore unsuitable for the construction of lawlike and hence inductively confirmable propositions. The predicate "green" on the other hand, is free from a reference to a specific time coordinate, and hence is suitable for the construction of lawlike, and hence inductively confirmable, propositions or hypotheses. Before we discuss this, let us look at another example, which shows that no nonsense was involved in the discussion so far.

The second example is formally analogous to the first one. But there is one big difference; it does not involve the introduction of a new hitherto non-existent predicate. Let $K(x)$ and $E(x)$ mean "x consists of copper" and "x is an electric conductor". Then on the basis of observed objects that consist of copper and that conduct electricity, the following hypothesis is positively confirmed, not violated, and not exhausted:

$$\forall x(K(x) \Longrightarrow E(x)) \quad \text{(All copper conducts electricity).} \tag{3}$$

Now the opposite hypothesis

$$\forall x(K(x) \Longrightarrow \neg E(x)) \quad \text{(All copper does not conduct electricity)} \tag{4}$$

is clearly violated on the basis of all the cases which confirm (3). Hence one would assume that for a new object d made of copper, but that has not yet been examined with respect to its conductivity of electricity, the proposition or prediction

$$E(d) \tag{5}$$

is well confirmed. One would justify this on the basis that d is made of copper, that is, $K(d)$ is true, and that (3), that is, "All copper conducts electricity" has received positive confirmation.

But we can also establish the opposite result, namely, that $\neg E(d)$ is also well confirmed. We introduce a further predicate $P(x)$.

Definition 1.2 $P(x) \stackrel{def}{=}$ *x was examined before the present time t_0 with respect to its ability to conduct electricity.*

We can now construct the following universal hypothesis:

$$\forall x(K(x) \Longrightarrow ((P(x) \wedge E(x)) \vee (\neg P(x) \wedge \neg E(x)))) \tag{6}$$

(Read: It holds for all copper objects that either they have been examined before t_0 and are conductors, or they have not been examined before t_0, and are non-conductors.)

For a positive single case (say a) of hypothesis (3), namely,

$$\forall x(K(x) \Longrightarrow E(x))$$

$K(a)$ and $E(a)$ must hold. But the positive single case a of "All copper conducts electricity" also satisfies the newly introduced predicate $P(x)$ (definition 1.2), because a can only be a positive single case of the hypothesis "All copper conducts electricity" if a was examined before the present time t_0 with respect to its ability to conduct electricity. Consequently, if a is a positive single case of the hypothesis "All copper conducts electricity" then given the definition of the new predicate $P(x)$ (definition 1.2) the following holds:

$$K(a) \wedge E(a) \wedge P(a). \tag{7}$$

But on the basis of the conjunction $K(a) \wedge E(a) \wedge P(a)$, the object a is also a positive single case of "All copper objects that either have been examined before t_0 and are conductors, or have not been examined before t_0, and are non-conductors". Moreover, if hypothesis "All copper conducts electricity" is not violated, then the hypothesis "All copper objects that either have been examined before t_0 and are conductors, or have not been examined before t_0, and are non-conductors", is also not violated.

In addition, the hypothesis "All copper objects that either have been examined before t_0 and are conductors, or have not been examined before t_0, and are non-conductors" is not exhausted, since not all objects satisfying its antecedent have been examined before t_0.

Now consider a new piece of copper d, which has not yet been examined. Hence

$$K(d) \wedge \neg P(d) \tag{8}$$

holds. On the basis of the hypothesis "All copper conducts electricity", which is positively confirmed, not violated and not exhausted we would conclude that this new item of copper conducts electricity, that is, $E(d)$. However, on the basis of the hypothesis "All copper objects that either have been examined before t_0 and are conductors, or have not been examined before t_0, and are non-conductors," we get $\neg E(d)$. That is,

1. $\forall x(K(x) \Longrightarrow ((P(x) \wedge E(x)) \vee (\neg P(x) \wedge \neg E(x))))$
2. $K(d) \wedge \neg P(d)$

$\therefore \quad \neg E(d)$

The hypothesis

$$\forall x(K(x) \Longrightarrow ((P(x) \wedge E(x)) \vee (\neg P(x) \wedge \neg E(x))))$$

was confirmed until t_0 through all the positive single cases that confirmed the hypothesis

$$\forall x(K(x) \Longrightarrow E(x)).$$

This shows that one obtains inconsistent results with valid and plausible inductive methods if one does not differentiate between lawlike and accidental propositions. In the present case it is hypothesis

$$\forall x(K(x) \Longrightarrow ((P(x) \wedge E(x)) \vee (\neg P(x) \wedge \neg E(x))))$$

which fails to be a lawlike proposition.

It is only with regard to new instances that we get inconsistent inductive results. And we get these inconsistent inductive results on the same set of positive single cases which confirm both hypotheses

$$\forall x(K(x) \Longrightarrow E(x))$$

and

$$\forall x(K(x) \Longrightarrow ((P(x) \wedge E(x)) \vee (\neg P(x) \wedge \neg E(x)))).$$

If d had been an item that was examined before t_0, then hypothesis

$$\forall x(K(x) \Longrightarrow ((P(x) \wedge E(x)) \vee (\neg P(x) \wedge \neg E(x))))$$

would have yielded the same results as hypothesis

$$\forall x(K(x) \Longrightarrow E(x)),$$

namely, that d is a conductor.

Goodman maintains that without a clear explication of the concept of lawlikeness no satisfactory solution to the problem of inductive confirmation of propositions can be expected.