Philosophy 352

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Contents

Part I

Monadic Quantification Logic Involving Non-multiple General Propositions ($\mathbf{Q}^-$)

1 Derivation Rules $\mathbf{Q}^-\mathbf{D}$

1.1 Derivation Rules of $\mathbf{SD}^+$

1.2 Quantification Derivation Rules

Remark 1.1 *It should be noted that all the Derivation Rules of QL presented below do not permit any open wffs to occur anywhere in the derivations. A variable is either bound* locally*, that is, the variable in question occurs within the scope of a quantifier* $\mathbb{JI}$ *of the wff in which it occurs, or the variable in question is bound* globally*, that is, the variable is in the scope of some* commonizing quantifier $\mathbb{JI}$. *Recall that* $\mathbb{JI} \in \{\forall, \exists\}$.

Remark 1.2 *We will use* u, v, w*, with or without subscripts, as meta-variables which range over the set of variable* $\{x, y, z; x_1, x_2, \ldots\}$.

1.2.1 Universal Commonization (UC)

$$
\begin{array}{l|l}
 & \vdots \\
\text{i} & \forall v \varphi_1(v) \\
 & \vdots \\
\text{k} & \forall v \varphi_k(v) \\
 & \quad \begin{array}{|ll}
 \forall v & \text{UC} \\ \hline
 \varphi_1(v) & \text{UC i} \\
 \vdots & \\
 \varphi_k(v) & \text{UC k} \\
 \vdots &
 \end{array} \\
 & \vdots
\end{array}
$$

Remark 1.3 *There need not be any* $\forall v \varphi_i(v)$*; that is, one may have* $k = 0$. *Hence*

$$
\begin{array}{l|l}
 & \vdots \\
 & \quad \begin{array}{|ll}
 \forall v & \mathit{UC} \\ \hline
 \vdots &
 \end{array} \\
 & \vdots
\end{array}
$$

1.2.2 Existential Commonization (EC)

$$
\begin{array}{l|l}
 & \vdots \\
\text{i} & \forall v \varphi_1(v) \\
 & \vdots \\
\text{j} & \forall v \varphi_k(v) \\
\ell & \exists v \varphi(v) \\
 & \vdots \\
 & \quad \begin{array}{|ll}
 \exists v & \text{EC} \\ \hline
 \varphi_1(v) & \text{EC i} \\
 \vdots & \\
 \varphi_k(v) & \text{EC j} \\
 \varphi(v) & \text{EC } \ell \\
 \vdots &
 \end{array} \\
 & \vdots
\end{array}
$$

Remark 1.4 *Existential Commonisation can be done only with respect to* **one** *existential quantifier.*

Remark 1.5 *There need not be any* $\varphi(v)$ *or* $\varphi_i(v)$ $(k = 0)$. *Hence,*

$$
\begin{array}{|l}
\vdots \\
\quad \begin{array}{|ll}
 \exists v & \mathit{UC} \\ \hline
 \vdots &
 \end{array} \\
\vdots
\end{array}
$$

1.2.3 Universal Decommonization (UD) & Existential Decommonization (ED)

$$
\begin{array}{l|ll}
\text{i} & \quad \begin{array}{|l} \forall v \\ \hline \vdots \end{array} & \text{UC} \\
\text{j} & \quad \begin{array}{|l} \varphi(v) \end{array} & \\
\text{k} & \forall v \varphi(v) & \text{UD i–j} \\
 & \vdots &
\end{array}
\qquad
\begin{array}{l|ll}
\text{i} & \quad \begin{array}{|l} \exists v \\ \hline \vdots \end{array} & \text{EC} \\
\text{j} & \quad \begin{array}{|l} \varphi(v) \end{array} & \\
\text{k} & \exists v \varphi(v) & \text{ED i–j} \\
 & \vdots &
\end{array}
$$

Universal-Existential Decommonization Conditions:

1. v need not in fact occur in φ. Note, if v does not occur we have vacuous de-quantification. (See vacuous de-quantification rules below.)

2. More than one wff may be decommonized.

1.2.4 Universal Vacuous Quantification (UVQ) & Existential Vacuous Quantification (EVQ)

$$\begin{array}{r|l l}
 & \vdots & \\
\text{i} & \varphi & \\
 & \vdots & \\
\text{j} & \forall v\varphi & \text{UVQ i} \\
 & \vdots &
\end{array}
\qquad
\begin{array}{r|l l}
 & \vdots & \\
\text{i} & \varphi & \\
 & \vdots & \\
\text{j} & \exists v\varphi & \text{EVQ i} \\
 & \vdots &
\end{array}$$

Universal & Existential Vacuous Quantification Condition: v does not occur freely in φ, that is, $v \notin FV[\varphi]$.

1.2.5 Universal Vacuous Dequantification (UVDQ) & Existential Vacuous Dquantification (EVDQ)

$$\begin{array}{r|l l}
 & \vdots & \\
\text{i} & \forall v\varphi & \\
 & \vdots & \\
\text{j} & \varphi & \text{UVDQ} \\
 & \vdots &
\end{array}
\qquad
\begin{array}{r|l l}
 & \vdots & \\
\text{i} & \exists v\varphi & \\
 & \vdots & \\
\text{j} & \varphi & \text{EVDQ} \\
 & \vdots &
\end{array}$$

Universal & Existential Vacuous Dequantification Condition: v does not occur freely in φ, that is, $v \notin FV[\varphi]$. Indeed, if v did occur freely in φ, then the quantification would not be vacuous.

1.2.6 Universal Instantiation (UI)

$$\begin{array}{r|l l}
 & \vdots & \\
\text{i} & \forall v\varphi(v) & \\
 & \vdots & \\
\text{j} & \varphi(t) & \text{UI j} \\
 & \vdots &
\end{array}$$

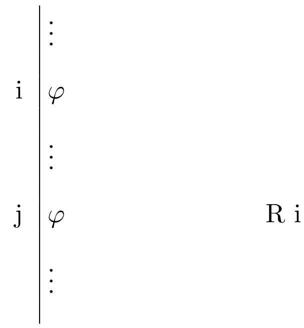
Universal Instantiation Condition: The term t is any individual constant or designator $\mathfrak{a}_i \in \mathcal{C}$ or $t = u$, where u is any variable (including $v = u$) provided it is captured by a commonizing quantifier $\text{Л}\!\!\text{I}u$ (or $\text{Л}\!\!\text{I}v$, if $t = v$).

1.2.7 Existential Generalization (EG)

$$\begin{array}{r|l l}
 & \vdots & \\
\text{i} & \varphi(t) & \\
 & \vdots & \\
\text{j} & \exists v\varphi(v) & \text{EG i} \\
 & \vdots &
\end{array}$$

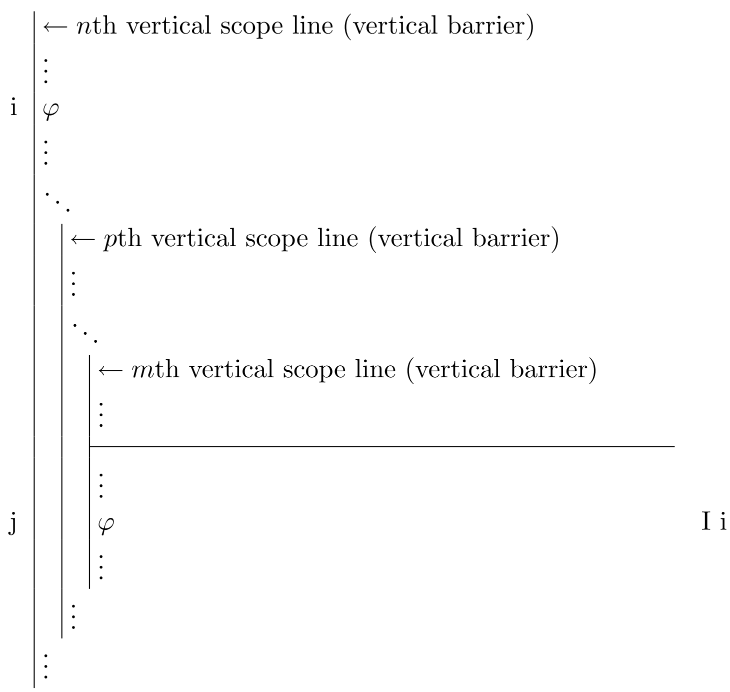
Existential Generalisation Condition: The term t is any individual constant $\mathfrak{a}_i \in \mathcal{C}$ or $t = v$.

1.2.8 Reiteration (R)



Reiteration Condition: A wff φ that occurs on a given line (i) to the immediate right of a vertical scope line (vertical barrier) of nth order, may be entered on a subsequent line $j > i$ provided it is immediately to the right of the *same unbroken* nth order vertical scope line. This rule is mainly used to make proofs clearer.

1.2.9 Importation (I)



Importation Condition: A wff φ that occurs on a horizontal line (i) to the right of a vertical scope line (vertical barrier) of order n may be entered on a subsequent line $j > i$ immediately to the right of a vertical scope line of order $m > n$ to the right of the scope line of order n, provided that no vertical scope line of order p, such that $m \geq p > n$, is crossed which is due to the quantifier commonization for a variable that occurs freely in φ.

2 Derivation Rules Q$^-$D$^+$

2.1 Derivation Rules Q^-D

2.2 Inference Rules

$$
\begin{array}{l|l|ll}
 & \vdots & & \\
\text{i} & & \text{Ⅱ}v & \\
\cline{3-3}
 & & \vdots & \\
\text{j} & & \varphi & \\
 & & \vdots & \\
\text{k} & & \neg\varphi & \\
\ell & & \bot & \text{j, k} \\
\text{m} & \bot & & \\
 & \vdots & &
\end{array}
\qquad
\begin{array}{l|l|ll}
 & \vdots & & \\
 & & \vdots & \\
\text{i} & & \varphi & \mathcal{H} \\
\cline{3-3}
 & & \vdots & \\
\text{j} & & \psi & \\
 & & \vdots & \\
\text{k} & & \neg\psi & \\
\ell & & \bot & \text{j, k} \\
\text{m} & \neg\varphi & & \neg\text{I} \\
 & \vdots & &
\end{array}
\qquad
\begin{array}{l|l|ll}
 & \vdots & & \\
 & & \vdots & \\
\text{i} & & \neg\varphi & \mathcal{H} \\
\cline{3-3}
 & & \vdots & \\
\text{j} & & \psi & \\
 & & \vdots & \\
\text{k} & & \neg\psi & \\
\ell & & \bot & \text{j, k} \\
\text{m} & \varphi & & \neg\text{E} \\
 & \vdots & &
\end{array}
$$

2.3 Replacement Rules

2.3.1 Quantifier Negation (QN)

$$
\begin{aligned}
\forall v\varphi(v) &\dashv\vdash \neg\exists v\neg\varphi(v)\\
\exists v\varphi(v) &\dashv\vdash \neg\forall v\neg\varphi(v)\\
\neg\forall v\varphi(v) &\dashv\vdash \exists v\neg\varphi(v)\\
\neg\exists v\varphi(v) &\dashv\vdash \forall v\neg\varphi(v)
\end{aligned}
$$

2.3.2 Quantifier Distribution (QD)

$$
\begin{aligned}
\forall v(\varphi(v) \wedge \psi(v)) &\dashv\vdash \forall v\varphi(v) \wedge \forall v\varphi(v)\\
\exists v(\varphi(v) \vee \psi(v)) &\dashv\vdash \exists v\varphi(v) \vee \exists v\psi(v)
\end{aligned}
$$

Suppose that $v \notin FV[\alpha]$.

$$
\begin{aligned}
\forall v(\alpha \wedge \varphi(v)) &\dashv\vdash (\alpha \wedge \forall v\varphi(v))\\
\exists v(\alpha \wedge \varphi(v)) &\dashv\vdash (\alpha \wedge \exists v\varphi(v))\\
\forall v(\alpha \vee \varphi(v)) &\dashv\vdash (\alpha \vee \forall v\varphi(v))\\
\exists v(\alpha \vee \varphi(v)) &\dashv\vdash (\alpha \vee \exists v\varphi(v))\\
\forall v(\alpha \implies \varphi(v)) &\dashv\vdash (\alpha \implies \forall v\varphi(v))\\
\exists v(\alpha \implies \varphi(v)) &\dashv\vdash (\alpha \implies \exists v\varphi(v))\\
\forall v(\varphi(v) \implies \alpha) &\dashv\vdash (\exists v\varphi(v) \implies \alpha)\\
\exists v(\varphi(v) \implies \alpha) &\dashv\vdash (\forall v\varphi(v) \implies \alpha)
\end{aligned}
$$

2.3.3 Other Useful Quantifier Replacement Laws (QR)

$$
\begin{aligned}
\neg\forall v(\varphi(v) \implies \psi(v)) &\dashv\vdash \exists v(\varphi(v) \wedge \neg\psi(v))\\
\neg\exists v(\varphi(v) \wedge \psi(v)) &\dashv\vdash \forall v(\varphi(v) \implies \neg\psi(v))
\end{aligned}
$$

3 Exercises in Monadic Quantification Logic

3.1 Part 1

Construct formal proofs of validity for the following arguments.

$$\begin{array}{rl} 1. & \forall x(A(x) \implies B(x)) \\ 2. & \neg B(t) \\ \hline \therefore & \neg A(t) \end{array} \tag{1}$$

$$\begin{array}{rl} 1. & \forall x(C(x) \implies D(x)) \\ 2. & \forall x(E(x) \implies \neg D(x)) \\ \hline \therefore & \forall x(E(x) \implies \neg C(x)) \end{array} \tag{2}$$

$$\begin{array}{rl} 1. & \forall x(F(x) \implies \neg G(x)) \\ 2. & \exists x(H(x) \wedge G(x)) \\ \hline \therefore & \exists x(H(x) \wedge \neg F(x)) \end{array} \tag{3}$$

$$\begin{array}{rl} 1. & \forall x(I(x) \implies J(x)) \\ 2. & \exists x(I(x) \wedge \neg J(x)) \\ \hline \therefore & \forall x(J(x) \implies I(x)) \end{array} \tag{4}$$

$$\begin{array}{rl} 1. & \forall x(K(x) \implies L(x)) \\ 2. & \forall x((K(x) \wedge L(x)) \implies M(x)) \\ \hline \therefore & \forall x(K(x) \implies M(x)) \end{array} \tag{5}$$

$$\begin{array}{rl} 1. & \forall x(N(x) \implies O(x)) \\ 2. & \forall x(P(x) \implies O(x)) \\ \hline \therefore & \forall x((N(x) \vee P(x)) \implies O(x)) \end{array} \tag{6}$$

$$\begin{array}{rl} 1. & \forall x(Q(x) \implies R(x)) \\ 2. & \exists x(Q(x) \vee R(x)) \\ \hline \therefore & R(x) \end{array} \tag{7}$$

$$\begin{array}{rl} 1. & \forall x(S(x) \implies (T(x) \implies U(x))) \\ 2. & \forall x(U(x) \implies (V(x) \wedge W(x))) \\ \hline \therefore & \forall x(S(x) \implies (T(x) \implies V(x))) \end{array} \tag{8}$$

$$
\begin{array}{ll}
1. & \forall x((X(x) \vee Y(x)) \implies (Z(x) \wedge A(x))) \\
2. & \forall x((Z(x) \vee A(x)) \implies (X(x) \wedge Y(x))) \\
\hline
\therefore & \forall x(X(x) \iff Z(x))
\end{array}
\tag{9}
$$

$$
\begin{array}{ll}
1. & \forall x((B(x) \implies C(x)) \wedge (D(x) \implies E(x))) \\
2. & \forall x[(C(x) \vee E(x)) \implies \{[F(x) \implies (G(x) \implies F(x))] \implies (B(x) \wedge D(x))\}] \\
\hline
\therefore & \forall x(B(x) \iff D(x))
\end{array}
\tag{10}
$$

3.2 Part 2

Construct formal proofs of validity for the following arguments. For the meta-variable v you may substitute x, y or z; that is, $v \in \{x, y, z\}$.

1. All athletes are brawny. Charles is not brawny. Therefore, Charles is not an athlete. (A(v), B(v), c)
2. No contractors are dependable. Some contractors are engineers. Therefore, some engineers are not dependable. (C(v), D(v), E(v))
3. All fiddlers are gay. Some hunters are not gay. Therefore, some hunters are not fiddlers. (F(v), E(v), H(v))
4. No judges are idiots. Kanter is an idiot. Therefore, Kanter is not a judge. (I(v), I(v), k)
5. All liars are mendacious. Some liars are newspapermen. Therefore, some newspapermen are mendacious. (L(v), M(v), N(v))
6. No osteopaths are pediatricians. Some quacks are pediatricians. Therefore, some quacks are not osteopaths. (O(v), P(v), Q(v))
7. Only salesmen are retailers. Not all retailers are travelers. Therefore, some salesmen are not travelers. (S(v), R(v), T(v))
8. There are no uniforms that are not washable. There are no washable velvets. Therefore, there are no velvet uniforms. (U(v), W(v), V(v))
9. Only authoritarians are bureaucrats. Authoritarians are all churlish. Therefore, any bureaucrat is churlish. (A(v), B(v), C(v))
10. Dates are edible. Only items of food are edible. All items of food are good. Therefore, all dates are good. (D(v), E(v), F(v), E(v))
11. All dancers are graceful. Mary is a student. Mary is a dancer. Therefore, some students are graceful. (D(v), E(v), S(v), m)
12. Tigers are fierce and dangerous. Some tigers are beautiful. Therefore, some dangerous things are beautiful. (T(v), F(v), D(v), B(v)) ,
13. Bananas and grapes are fruits. Fruits and vegetables are nourishing. Therefore, bananas are nourishing. (B(v), E(v), F(v), V(v), N(v))
14. A communist is either a fool or a knave. Fools are naive. Not all communists are naive. Therefore, some communists are knaves. (C(v), F(v), K(v), N(v))

15. All butlers and valets are both obsequious and dignified. Therefore, all butlers are dignified. (B(v), V(v), O(v), D(v))

16. All houses built of brick are warm and cozy. All houses in Englewood are built of brick. Therefore, all houses in Englewood are warm. (H(v), B(v), W(v), C(v), E(v))

17. All professors are learned. All learned professors are savants. Therefore, all professors are learned savants. (P(v), L(v), S(v))

18. All diplomats are public servants. Some diplomats are eloquent. All eloquent public servants are orators. Therefore, some diplomats are orators. (D(v), P(v), E(v), O(v))

19. Doctors and lawyers are college graduates. Any altruist is an idealist. Some lawyers are not idealists. Some doctors are altruists. Therefore, some college graduates. are idealists. (D(v), L(v), C(v), A(v), I(v))

20. Bees and wasps sting if they are either angry or frightened. Therefore, any bee stings if it is angry. (B(v), W(v), S(v), A(v), F(v))

21. Any authors are successful if and only if they are well read. All authors are intellectuals. Some authors are successful but not well read. Therefore, all intellectuals are authors. (A(v), S(v), W(v), I(v))

22. Every passenger is either in first class or in tourist class. Each passenger is in tourist class if and only if he is not wealthy. Some passengers are wealthy. Not all passengers are wealthy. Therefore, some passengers are in tourist class. (P(v), F(v), T(v), W(v))

23. All members are both officers and gentlemen. All officers are fighters. Only a pacifist is either a gentleman or not a fighter. No pacifists are gentlemen if they are fighters. Some members are fighters if and only if they are officers. Therefore, not all members are fighters. (M(v), O(v), G(v), F(v), P(v))

24. Wolfhounds and terriers are hunting dogs. Hunting dogs and lap dogs are domesticated animals. Domesticated animals are gentle and useful. Some wolfhounds are neither gentle nor small. Therefore, some terriers are small but not gentle. (W(v), T(v), H(v), L(v), O(v), G(v), U(v), S(v))

25. No man who is a candidate will be defeated if he is a good campaigner. Any man who runs for office is a candidate. Any candidate who is not defeated will be elected. Every man who is elected is a good campaigner. Therefore, any man who runs for office will be elected if and only if he is a good campaigner. (M(v), E(v), O(v), G(v), R(v), E(v))