Philosophy 352
Introduction to Symbolic Logic
Second Midterm

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Part One

Provide a formal proof of the following syntactic equivalence, without using the replacement rules of quantifier negation. (4 points)

1. $\neg\forall x F(x) \dashv\vdash \exists x \neg F(x)$

Part Two

Provide a formal proof of each of the following theorems (4 points each):

2. $\emptyset \vdash (\forall x P(x) \vee \forall x Q(x)) \Longrightarrow \forall x(P(x) \vee Q(x))$
3. $\emptyset \vdash \exists x(\exists x P(x) \Longrightarrow P(x))$
4. $\emptyset \vdash \neg\exists x P(x) \Longrightarrow \forall x(P(x) \Longrightarrow Q(x))$

Part Three

Each of the following arguments is valid. Construct a formal proof for each of them, in each case using the suggested notation.

5. Whatever is either a worker or a queen is a female. Everything that is either a female or a builder is a bee. Hence, all workers are bees. ($W(x)$: x is a worker; $Q(x)$: x is a queen; $F(x)$: x is a female; $U(x)$: x is a builder; $B(x)$: x is a bee) (6 points)
6. A book is interesting only if it is well written. A book is well written only if it is interesting. Therefore any book is both interesting and well written if it is either interesting or well written. ($B(x)$: x is a book; $I(x)$: x is interesting; $W(x)$: x is well written) (8 points)

$\perp$ -

#1 $\vdash$

1	$\neg \forall x\, F(x)$ /∴ $\exists x \neg F(x)$	
2	$\neg \exists x \neg F(x)$ /∴ $\perp$	UC
3	$\forall x$	
4	$\neg F(x)$	EG 4
5	$\exists x \neg F(x)$	2, 5
6	$\perp$	$\neg$E 4-6
7	$F(x)$	UDC 3-7
8	$\forall x\, F(x)$	1, 8
9	$\perp$	$\neg$E, 2-9
10	$\exists x \neg F(x)$	

1	$\exists x \neg F(x)$ /∴ $\neg \forall x\, F(x)$	
2	$\forall x\, F(x)$ /∴ $\perp$	
3	$\exists x$	EC
4	$\neg F(x)$	EC 1
5	$F(x)$	EC 2
6	$\perp$	4, 5
7	$\perp$	$\neg$I, 2-7
8	$\neg \forall x\, F(x)$	

ϕ $/\therefore (\forall x\, P(x) \vee \forall x\, Q(x)) \Rightarrow \forall x\, (P(x) \vee Q(x))$

#2

1	$\forall x\, P(x) \vee \forall x\, Q(x)$	$/\therefore \forall x\, (P(x) \vee Q(x))$
2	$\forall x\, P(x)$	
3	$\forall x$	UC
4	$P(x)$	UC 2
5	$(P(x) \vee Q(x))$	$\vee$I, 4
6	$\forall x\, (P(x) \vee Q(x))$	UDC 3-5
7	$\forall x\, Q(x)$	
8	$\forall x$	UC
9	$Q(x)$	UC 7
10	$(P(x) \vee Q(x))$	$\vee$I 9
11	$\forall x\, (P(x) \vee Q(x))$	UDC 8-10
12	$\forall x\, (P(x) \vee Q(x))$	$\vee$E, 1, 2-6, 7-11
	$(\forall x\, P(x) \vee \forall x\, Q(x)) \Rightarrow \forall x\, (P(x) \vee Q(x))$	$\Rightarrow$I, 1-12

#3

$\phi \;\; /\therefore\; \exists x\,(\exists x\, P(x) \Rightarrow P(x))$

1	$\neg \exists x\,(\exists x\, P(x) \Rightarrow P(x))$	
2	$\forall x\, \neg(\exists x\, P(x) \Rightarrow P(x))$	
3	$\forall x\,(\exists x\, P(x) \wedge \neg P(x))$	
4	$\forall x$	UC
5	$\exists x\, P(x) \wedge \neg P(x)$	UC 3
6	$\exists x\, P(x)$	$\wedge$E 5
7	$\neg P(x)$	$\wedge$E 5
8	$\exists x\, P(x)$	VUDC 6
9	$\forall x\, \neg P(x)$	UDC 7
10	$\exists x$	EC
11	$P(x)$	EC 8
12	$\neg P(x)$	EC 9
13	$\bot$	11, 12
14	$\bot$	
15	$\exists x\,(\exists x\, P(x) \Rightarrow P(x))$	$\neg$E, 1–14

Another way that is shorter

8	$\exists x\, P(x)$	
9	$\forall x\, \neg P(x)$	
10	$\neg \exists x\, P(x)$	QN 9
	$\bot$	8, 10

⋮

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	$\emptyset \quad /\therefore\ \neg \exists x\, P(x) \Rightarrow \forall x\,(P(x) \Rightarrow Q(x))$	
1	$\neg \exists x\, P(x) \quad /\therefore\ \forall x\,(P(x) \Rightarrow Q(x))$	
2	$\forall x$	UC
3	$P(x) \quad /\therefore\ Q(x)$	
4	$\exists x\, P(x)$	EG 3
5	$\perp$	1, 5
6	$Q(x)$	$\perp$ E
7	$(P(x) \Rightarrow Q(x))$	$\Rightarrow$I, 3-6
8	$\forall x\,(P(x) \Rightarrow Q(x))$	UDC 2-7
9	$\neg \exists x\, P(x) \Rightarrow \forall x\,(P(x) \Rightarrow Q(x))$	$\Rightarrow$I, 1-8

5

1	$\forall x\,[(W(x) \vee Q(x)) \Rightarrow F(x)]$	
2	$\forall x\,[(F(x) \vee U(x)) \Rightarrow B(x)]$	$/\therefore \forall x\,(W(x) \Rightarrow B(x))$
3	$\forall x$	UC
4	$(W(x) \vee Q(x)) \Rightarrow F(x)$	UC 1
5	$(F(x) \vee U(x)) \Rightarrow B(x)$	UC 2
6	$W(x)$ $/\therefore B(x)$	
7	$(W(x) \vee Q(x))$	$\vee$I, 6
8	$F(x)$	$\Rightarrow$E 4,8
9	$(F(x) \vee U(x))$	$\vee$I 8
10	$B(x)$	$\Rightarrow$E, 5,8
11	$(W(x) \Rightarrow B(x))$	$\Rightarrow$I, 6-10
12	$\forall x\,(W(x) \Rightarrow B(x))$	UDC 3-11

#6

1	$\forall x[(B(x) \Rightarrow (I(x) \Rightarrow W(x)))]$	
2	$\forall x[(B(x) \Rightarrow (W(x) \Rightarrow I(x)))]$ $/\therefore$ $\forall x[B(x) \Rightarrow ((I(x) \vee W(x)) \Rightarrow (I(x) \wedge W(x)))]$	
3	$\forall x$	UC
4	$B(x) \Rightarrow (I(x) \Rightarrow W(x))$	UC 1
5	$B(x) \Rightarrow (W(x) \Rightarrow I(x))$	UC 2
6	$B(x)$ $/\therefore$ $(I(x) \vee W(x)) \Rightarrow (I(x) \wedge W(x))$	
7	$I(x) \Rightarrow W(x)$	$\Rightarrow$ E, 4, 6
8	$W(x) \Rightarrow I(x)$	$\Rightarrow$ E, 5, 6
9	$(I(x) \vee W(x))$ $/\therefore$ $(I(x) \wedge W(x))$	
10	$I(x)$	
11	$W(x)$	$\Rightarrow$ E, 7, 10
12	$I(x) \wedge W(x)$	$\wedge$ I, 10, 11
13	$W(x)$	
14	$I(x)$	$\Rightarrow$ E, 8, 13
15	$I(x) \wedge W(x)$	$\wedge$ I, 13, 14
16	$(I(x) \wedge W(x))$	$\vee$ E 9, 10–12, 13–15
17	$(I(x) \vee W(x)) \Rightarrow (I(x) \wedge W(x))$	$\Rightarrow$ I, 9–16
18	$B(x) \Rightarrow ((I(x) \vee W(x)) \Rightarrow (I(x) \wedge W(x)))$	$\Rightarrow$ I, 6–17
19	$\forall x[B(x) \Rightarrow ((I(x) \vee W(x)) \Rightarrow (I(x) \wedge W(x)))]$	UDC 3–18