Philosophy 352: Exercises using $\Longrightarrow$ introduction/elimination rules and the importation/exportation rules only

Dr. H. Korté
Department of Philosophy
University of Regina

January 16, 2012

1 Part 1

1. $\{(C \Longrightarrow B), (A \Longrightarrow C)\} \vdash (A \Longrightarrow B)$

2. $\{(A \Longrightarrow (C \Longrightarrow (B \Longrightarrow C)))\} \vdash (C \Longrightarrow (A \Longrightarrow (B \Longrightarrow C)))$

3. $\{B\} \vdash (A \Longrightarrow (A \Longrightarrow A))$

4. $\{(B \Longrightarrow C)\} \vdash (A \Longrightarrow A)$

5. $\{((A \Longrightarrow (B \Longrightarrow A)) \Longrightarrow C)\} \vdash C$

6. $\{((P \Longrightarrow Q) \Longrightarrow Q), (P \Longrightarrow R)\} \vdash ((R \Longrightarrow Q) \Longrightarrow Q)$

7. $\{((P \Longrightarrow Q) \Longrightarrow Q), (Q \Longrightarrow R), (R \Longrightarrow Q)\} \vdash ((P \Longrightarrow R) \Longrightarrow R)$

8. $\{P, (((Q \Longrightarrow Q) \Longrightarrow P) \Longrightarrow R)\} \vdash R$

9. $\{((P \Longrightarrow Q) \Longrightarrow (R \Longrightarrow Q))\} \vdash ((P \Longrightarrow Q) \Longrightarrow (R \Longrightarrow R))$

10. $\{((P \Longrightarrow Q) \Longrightarrow P), (P \Longrightarrow (R \Longrightarrow Q)), R\} \vdash P$

11. $\{((P \Longrightarrow R) \Longrightarrow ((Q \Longrightarrow R) \Longrightarrow (R \Longrightarrow P)))\} \vdash (R \Longrightarrow P)$

12. $\{(((R \Longrightarrow Q) \Longrightarrow (Q \Longrightarrow P)) \Longrightarrow (P \Longrightarrow Q))\} \vdash (P \Longrightarrow Q)$

2 Part 2

Find the categorical derivatins of the following wffs. A categorical derivation of φ is a derivation from the empty set, that is, $\emptyset \vdash \varphi$.

1. $((P \Longrightarrow Q) \Longrightarrow ((Q \Longrightarrow R) \Longrightarrow (P \Longrightarrow R)))$

2. $(R \Longrightarrow ((P \Longrightarrow Q) \Longrightarrow (S \Longrightarrow (P \Longrightarrow Q))))$

3. $(((P \Longrightarrow P) \Longrightarrow P) \Longrightarrow P)$

4. $((P \Longrightarrow (P \Longrightarrow Q)) \Longrightarrow (P \Longrightarrow Q))$

5. $(((P \Longrightarrow (Q \Longrightarrow R)) \Longrightarrow S) \Longrightarrow (R \Longrightarrow S))$

6. $(((P \implies P) \implies Q) \implies Q)$

7. $((P \implies Q) \implies (((R \implies Q) \implies S) \implies ((R \implies P) \implies S)))$